Dimensionless Numbers in Fluid Mechanics/Heat Transfer and Their Significance

Dimensionless Number	Formula	Significance	
Reynolds Number, Re =	$\frac{duL}{\mu} = \frac{uL}{v}$	$\frac{Inertial\ Forces}{Viscous\ Forces}$	•Ratio of Inertial forces to viscous forces. •Primarily used to analyze different flow regimes namely Laminar, Turbulent, or both. •When Viscous forces are dominant it's a laminar flow & when Inertial forces are dominant it is a Turbulent flow.
Prandtl Number, Pr =	$\frac{c_p\mu}{k} = \frac{\nu}{\alpha}$	$\frac{Momentum\ Diffusivity}{Heat\ Diffusivity}$	•Depends only on fluid & its properties. It is also ratio of velocity boundary layer to thermal boundary layer •Pr = small, implies that rate of thermal diffusion (heat) is more than the rate of momentum diffusion (velocity). •Also the thickness of thermal boundary layer is much larger than the velocity boundary layer.

Prandtl Number	Heat Transfer Condition	Correlation
Pr < 0.1 (e.g. liquid metals)	Constant Heat Rate	$Nu = 6.3 + .003(RePr)$
Pr < 0.1 (e.g. liquid metals)	Constant Temperature	$Nu = 4.8 + .003(RePr)$
0.5 < Pr < 1 (e.g. gases)	Constant Heat Rate	$Nu = 0.022(Re^{0.8}\ Pr^{0.6})$
0.5 < Pr < 1 (e.g. gases)	Constant Temperature	$Nu = 0.021(Re^{0.8}\ Pr^{0.6})$
1.0 < Pr < ~20 (e.g. water & light liquids)	All	$Nu = 0.0155(Re^{0.83}\ Pr^{0.5})$
Pr > ~20 (e.g. oils & viscous liquids)	All	$Nu = 0.0118(Re^{0.9}\ Pr^{0.3})$

$$Pr^{1/3} = \frac{\delta}{\delta_t}$$

Dimensionless Numbers in Fluid Mechanics/Heat Transfer and Their Significance – Cont'd

Dimensionless Number	Formula	Significance	
Schmidt Number, Sc =	$\frac{\mu}{\rho D_{AB}}$	$\frac{Momentum\ Diffusivity}{Mass\ Diffusivity}$	Analogous of Prandtl number in Heat Transfer. •Used in fluid flows in which there is simultaneous momentum & mass diffusion. •It is also ratio of fluid boundary layer to mass transfer boundary layer thickness. •To find mass transfer coefficient using Sherwood number, we need Schmidt number.
Euler Number, Eu =	$\frac{\Delta P}{\rho V^2}$	$\frac{Pressure\ Energy}{Kinetic\ Energy}$	Used in fluid flow calculations where local pressure drop is necessary (dp = upstream pressure - downstream pressure) •Used to characterize the losses in the flow. •Eu = 1 corresponds to a perfect frictionless fluid flow.
Cavitation Number, Ca =	$\frac{P - P_v}{\frac{1}{2}\rho V^2}$	$\frac{local\ absolute\ P - vapor\ P}{KE}$	It gives the possibility / potential of a fluid to cavitate. •If Ca < 0, Cavitation occurs & if Ca > 0 no cavitation will occur, since the condition to avoid cavitation is that the minimum pressure (Pmin)within the entire pump should be greater than the vapor pressure (Pv) of the fluid at that temperature. (Pmin > Pv)

Dimensionless Numbers in Fluid Mechanics/Heat Transfer and Their Significance – Cont'd

Dimensionless Number	Formula	Significance	
Froude Number, Fr =	$\frac{V}{\sqrt{gL}}$	$\frac{\text{Inertial Forces}}{\text{Gravity Forces}}$	It is the ratio of mean flow velocity to the speed of small gravity wave along the water surface. •It is an indication of resistance to partially submerged object moving through to water. •Greater Fr value, greater is the resistance to flow. •Fr < 1 indicates subcritical flow (tranquil flow) •Fr > 1 indicates supercritical flow (rapid flow) •Fr = 1 indicates critical flow. •Used in ship design i.e. to analyze water flow around ships. •Inverse of the square of Fr is called Richardson Number (importance of natural convection to forced convection)
Mach Number, M or Ma =	$\frac{V}{C}$	$\frac{\text{Gas Velocity}}{\text{Speed of Sound}}$	To check whether the fluid can be considered compressible or not. •If M < 0.2-0.3, then the fluid medium can be considered steady & isothermal & hence incompressible. •Used for fluids flowing with high speeds in channels, nozzles, diffusers etc. •It is analogous to Froude Number C = speed of sound = 345m/s (at 15 deg. Celsius temperature)

Mach Number	Regime
< 1	Subsonic
= 1	Sonic
0.8 - 1.2	Transonic
1.2 – 5	Supersonic
5 - 10	Hypersonic
> 10	High - Hypersonic

Dimensionless Number	Formula	Significance	
Cauchy Number, C =	$\frac{\rho V^2}{K}$	$\frac{Inertial\ Forces}{Compressible\ Forces}$	It is the square of Mach number (Mach number can also be expressed in terms of bulk modulus as the square root of Cauchy number number). •Used to study compressible flow. K = bulk modulus of elasticity
(A)*Fanning Friction Factor, f =*	$\frac{\tau_W}{\frac{1}{2}\rho V^2}$	$\frac{Wall\ Stress}{Momentum\ Flux}$	•Used to study fluid friction in pipes.
(B) *Fanning Friction Factor, f =*	$\frac{e_f\ D}{2V^2L}$	$\frac{Energy\ Dissipated}{KE\ of\ Flow}\ X\ \frac{4L}{D}$	Tw = wall stress ef = friction loss Darcy Friction Factor (f_D) =4f
Weber Number, We =	$\frac{\rho V^2 L}{\sigma}$	$\frac{Inertia\ Force}{Surface\ Tension\ Force}$	•Useful in analyzing fluid flows where there is an interface between two different fluids, especially for multiphase flows with strongly curved surface. • The quantity is useful in analyzing thin film flows and the formation of droplets and bubbles.

Dimensionless Numbers in Fluid Mechanics/Heat Transfer and Their Significance – Cont'd

Dimensionless Number	Formula	Significance	
Pressure Coefficient, C_P =	$\frac{\Delta P}{(\frac{\rho u^2}{2})}$	$\frac{Pressure\ Force}{Inertial\ Force}$	Describes the relative pressures throughout a flow field in fluid dynamics.
Drag Coefficient, C_D =	$\frac{F_D}{(\frac{\rho u^2}{2})}$	$\frac{Total\ Drag\ Force}{Inertial\ Force}$	Used to quantify the drag or resistance of an object in a fluid environment, such as air or water.
Lift Coefficient, C_L =	$\frac{L}{qS}$	$\frac{Lift\ Force}{Inertial\ Force}$	Relates the lift generated by a lifting body to the fluid density around the body, the fluid velocity and an associated reference area.
Stokes Number, Stk or S_K =	$\frac{\tau U_0}{d_c}$		Commonly used in particles suspended in fluid. •For *Stk* << 1, the particle negotiates the obstacle. •For *Stk* >> 1, the particle travels in straightline and eventually collides with obstacle.
Eckert Number, E_c =	$\frac{u^2}{c_p \Delta T}$	$\frac{KE}{Enthalpy}$	Eckert number represents the kinetic energy of the flow relative to the boundary layer enthalpy difference. *Ec* plays an important role in high speed flows for which viscous dissipation is significant
Graetz number, Gz =	$\frac{D_H}{L} RePr$		Characterizes laminar flow in a conduit •A Graetz number of approximately 1000 or less is the point at which flow would be considered thermally fully developed.
Grashof number, Gr_L =	$\frac{g\beta(T_s - T_\infty)L^3}{\nu^2}$	$\frac{Bouyancy}{Viscous\ Force}$	Heat transfer=>natural convection.
Knudsen number, Kn =	$\frac{\lambda}{L}$	$\frac{Mean\ Free\ Path}{Length}$	Ratio of gas molecule mean free path to process length scale Indicates validity of line of sight (> 1) or continuum (< 0.01) gas models

Dimensionless Numbers in Fluid Mechanics/Heat Transfer and Their Significance – Cont'd

Dimensionless Number	Formula	Significance	
Bond Number, Bo =	$\frac{\rho g L^2}{\sigma}$	$\frac{Bouyant\ Force}{Capillary\ Force}$	Measuring the importance of surface tension forces compared to body forces and is used (together with Morton number) to characterize the shape of bubbles or drops moving in a surrounding fluid.
Morton Number, Mo =	$\frac{g\mu^4}{\Delta\rho\sigma^3} = \frac{We^3}{FrRe^4}$		Used together with the Bond number to characterize the shape of bubbles or drops moving in a surrounding fluid or continuous phase, c.
Peclet Number, Pe =	**Heat transfer:** **$Re_L Pr$ =** $\frac{Lu}{\alpha}$	$\frac{advective\ transport\ rate}{diffusive\ transport\ rate}$	Ratio of the rate of advection of a physical quantity by the flow to the rate of diffusion of the same quantity driven by an appropriate gradient. The thermal Peclet number is equivalent to the product of the Reynolds number and the Prandtl number.
Rayleigh Number, Ra =	**GrPr =** $\frac{g\beta(T_{hot}-T_{ref})L^3}{\nu\alpha}$	$\frac{buoyancy}{viscous\ x\ rate\ of\ heat\ diffussion}$	When the Rayleigh number is below a critical value for that fluid, heat transfer is primarily in the form of conduction; when it exceeds the critical value, heat transfer is primarily in the form of convection.
Bejan number, Be =	$\frac{\Delta P L^2}{\mu\upsilon}$		Dimensionless pressure drop along a channel of length L. **υ** = momentum diffusivity In the context of heat transfer **υ** is replaced by **α** the thermal diffusivity.

Dimensionless Numbers in Fluid Mechanics/Heat Transfer and Their Significance – Cont'd			
Dimensionless Number	**Formula**	**Significance**	
Hagen, Hg =	$-\frac{dP}{dx}\frac{\rho L^3}{\mu^2}$		It is the forced flow equivalent of Grashof number.
Nusselt, Nu =	$\frac{hd}{k}$	$\frac{Convection}{Conduction}$	Nusselt number represents the dimensionless temperature gradient at the solid surface.
Biot, Bi =	$\frac{hL}{k_b}$	$\frac{\text{conductive resistance within the object}}{\text{convective heat transfer resistance across the object's boundary}}$	Used in unsteady state (transient) heat transfer conditions. •ratio of heat transfer resistance inside the body to heat transfer resistance at the surface of the body. OR ratio of internal thermal resistance to external thermal resistance. •Shows the variation of temperature inside the body w.r.t to time. •Bi < 0.1 => heat transfer resistance inside the body is very low => inside the body conduction takes place faster compared to convection at the surface. => no temperature gradient inside the body (uniformity in temperature) vice versa implies that Temperature is not uniform throughout the material volume.

Dimensionless Numbers in Fluid Mechanics/Heat Transfer and Their Significance – Cont'd

Nomenclature:

μ => fluid absolute viscosity	**k => thermal conductivity**
v => fluid kinematic viscosity	**c_P => specific heat**
σ => surface tension	**g => gravitational acceleration**
ρ => fluid density	**h => heat transfer coefficient**
Δρ => difference in density of the two phases	**D_{AB} => mass diffusivity coefficient**
α => thermal diffusivity	**u, V => characteristic velocity scale**
υ => momentum diffusivity	**c => speed of sound**
λ => mean free path	**S => relevant plan area**
L => length	**q => fluid dynamic pressure**
d => diameter	**F_D => total drag force**
δ => velocity boundary layer	**β => volumetric thermal expansion coefficient**
δ_t => thermal boundary layer	**ΔT => characteristic temperature difference**
P => pressure	**T_∞ => quiescent temperature of the fluid**
ΔP => characteristic pressure difference of flow	**T_s => surface temperature**
P_v => vapor pressure	**T_{hot}=> temperature of the hot wall**
$\frac{dP}{dx}$ => pressure gradient	**T_{ref} => reference/surface temperature**

References and Sources

Books

Applied Fluid Dynamics Handbook by Robert D. Blevins
Fluid Mechanics by Frank White
Fox and McDonald's Introduction to Fluid Mechanics by Philip J. Pritchard , John W. Mitchell
Viscous Fluid Flow by Frank White

Websites

eFluids.com
Wikipedia.org /wiki/Dimensionless_numbers_in_fluid_mechanics

Contact Info:

Mig-Tech Fluidics Design
Principal: John Migniuolo
Email: john@fluidicsdesign.com
Phone: 914-241-2048